WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
परीक्षार्थी प्रवेश–पत्र के अनुसार भरें

विषय Subject : MATHEMATICS

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : THURSDAY, 20/03/2014

उत्तर देने का माध्यम
Medium of answering the paper : ENGLISH

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाए
Write Code No. as written on the top of Question Paper : 65/2

अतिरिक्त उत्तर–पुस्तिका (ओं) की संख्या
No. of Supplementary answer-book(s) used : 1

किसी शारीरिक अक्षमता के प्रमाणित हो तो संबंधित वर्ग में ✓ का निशान लगाएं।
If Physically challenged, tick the category

B	D	H	S	C

B = दृष्टिहीन, D = मूक एवं बधिर, H = शारीरिक रुप से विकलांग, S = स्पास्टिक, C = डिस्लेक्सिक
B = Blind, D = Deaf & Dumb, H = Physically Handicapped, S = Spastic, C = Dyslexic

क्या लेखन – लिपिक उपलब्ध करवाया गया : हाँ/नहीं
Whether writer provided : Yes / No —

*एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।

Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए
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केंद्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
Central Board of Secondary Education, Delhi

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
SENIOR SCHOOL CERTIFICATE EXAMINATION (CLASS XII)

प्रमाणित किया जाता है मैने/हमने इस उत्तर पुस्तिका का मूल्यांकन प्रश्न पत्र के समुचित सेट के अनुसार और पूर्ण रूप से मूल्यांकन पद्धति के अनुसार किया है।

Certified that I/We have evaluated this answer-book according to the correct set of question paper and strictly as per the marking scheme.

CB

Section - C

24. Let $P(E_1)$ be the probability of a two headed coin to be chosen,
$P(E_2)$ be the probability of a biased coin showing 75% times head to be chosen
and $P(E_3)$ be probability of a coin showing 40% ~~Hale~~ tails be chosen.

Let 'A' be the event of the tossed coin showing heads.

E_1, E_2, E_3 are mutually exclusive and exhaustive events.

By Baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)} \longrightarrow (1)$$

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$P\left(\frac{A}{E_1}\right) = 1$ [2 headed coin]

$P\left(\frac{A}{E_2}\right) = \frac{75}{100} = \frac{3}{4}$ [75% heads]

$P\left(\frac{A}{E_3}\right) = \frac{60}{100} = \frac{3}{5}$ [it shows 40% tails ∴ 60% heads]

$\therefore P\left(\frac{E_1}{A}\right) = \dfrac{\frac{1}{3}(1)}{\frac{1}{3}(1) + \frac{1}{3}\left(\frac{3}{4}\right) + \frac{1}{3}\left(\frac{3}{5}\right)}$ [using Baye's theorem (1)]

$= \dfrac{1}{1 + \frac{3}{4} + \frac{3}{5}} = \dfrac{1}{\frac{20+15+12}{20}}$

$= \dfrac{20}{20+27}$

$= \dfrac{20}{47}$

$\therefore$ Probability that it was the two headed coin when the randomly tossed coin showed head $= \cancel{\frac{20}{47}}$ $\left[P\left(\frac{E_1}{A}\right) \right]$

$= \frac{20}{47}$

25. Let π_1 be $x + y + z = 1$

π_2 be $2x + 3y + 4z = 5$

Let the equation of the plane passing through the line of intersection of both planes be π.

$$\pi = \pi_1 + \lambda \pi_2$$

$$\therefore \pi = (x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$\therefore (x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + 4\lambda) - 1 - 5\lambda = 0$$

$(1 + 2\lambda)$, $(1 + 3\lambda)$ and $(1 + 4\lambda)$ are the direction ratios of the plane π.

$\because$ The plane (π) is $\perp r$ to $[\pi_3 = x - y + z = 0]$
sum of product of direction ratios is zero.

$$\therefore (1+2\lambda)(1) - 1(1+3\lambda) + 1(1+4\lambda) = 0$$

$$\therefore 1 + 2\lambda + \cancel{\lambda} + 4\lambda = \cancel{\lambda} + 3\lambda$$

$$1 = 3\lambda - 6\lambda$$

$$= -3\lambda$$

$$\lambda = -1/3$$

$$\therefore \pi : x(1+2\lambda) + y(1+3\lambda) + z(1+4\lambda) - 1 - 5\lambda = 0$$

$$\Rightarrow x\left(1 - \frac{2}{3}\right) + y\left(1 - \frac{3}{3}\right) + z\left(1 - \frac{4}{3}\right) - 1 + \frac{5}{3} = 0$$

$$x\left(\frac{+1}{3}\right) + y(0) + z\left(\frac{-1}{3}\right) + \frac{2}{3} = 0$$

$$\cancel{x + x} \quad x - z + 2 = 0$$

$\therefore$ The equation of the required plane is $x - z + 2 = 0$

Distance of a point (x_1, y_1, z_1) from a plane $Ax + By + Cz = D$ is given by :

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A_1^2 + B_1^2 + C_1^2}} \right|$$

$\therefore$ Distance from the origin $= \left| \frac{0 + 0 + 0 + 2}{\sqrt{1+1}} \right|$

$$= \frac{2}{\sqrt{2}}$$

$$= \sqrt{2} \text{ units}$$

26. ~~Let~~ The amounts awarded for sincerity, truthfulness and helpfulness are Rs x, Rs y and Rs z respectively.

[awarded by schools A and B]

The equations are :-

$$3x + 2y + z = 1600$$
$$4x + y + 3z = 2300$$
$$x + y + z = 900$$

$$\underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}}_{X} = \underbrace{\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}_{3\times1}}_{B}$$

$$\therefore \quad X = A^{-1} B$$

$$|A| = 3(1-3) - 2(4-3) + 1(4-1)$$
$$= 3(-2) - 2 + 3$$
$$= -6 - 2 + 3$$
$$= -8 + 3$$
$$= \cancel{-7} \; -5$$

$|A| \neq 0 \quad \therefore \; A^{-1}$ exists.

$\therefore$ The equation of the required plane is $x - z + 2 = 0$

Distance of a point (x_1, y_1, z_1) from a plane $Ax + By + Cz = D$ is given by :

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A_1^2 + B_1^2 + C_1^2}} \right|$$

$\therefore$ Distance from the origin $= \left| \dfrac{0 + 0 + 0 + 2}{\sqrt{1+1}} \right|$

$$= \frac{2}{\sqrt{2}}$$

$$= \sqrt{2} \text{ units}$$

26. ~~Let~~ The amounts awarded for sincerity, truthfulness and helpfulness are Rs x, Rs y and Rs z respectively.
[awarded by schools A and B]

The equations are :-

$$3x + 2y + z = 1600$$
$$4x + y + 3z = 2300$$
$$x + y + z = 900$$

$$\underset{A}{\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}_{3\times 3}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1}} = \underset{B}{\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}_{3\times 1}}$$

$$\therefore \quad X = A^{-1} B$$

$$|A| = 3(1-3) - 2(4-3) + 1(4-1)$$
$$= 3(-2) - 2 + 3$$
$$= -6 - 2 + 3$$
$$= -8 + 3$$
$$= \cancel{-7} \; -5$$

$|A| \neq 0 \quad \therefore \; A^{-1}$ exists.

$\therefore X = A^{-1} B$

$$A^{-1} = \frac{adj.(A)}{|A|}$$

let A_{ij} represent the cofactors of elements in A.

$A_{11} = -2$

$A_{12} = -1$

$A_{13} = 3$

$A_{21} = -1$ $\therefore$ adj (A) = (Transpose of co-factor matrix)

$A_{22} = 2$

$A_{23} = -1$

$A_{31} = 5$

$A_{32} = -5$ $\therefore$ adj (A) = (co-factor matrix)'

$A_{33} = -5$

$$\text{co-factor maxtrix} = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$\therefore\ A^{-1} = \dfrac{\text{adj}(A)}{|A|}$ $[A \neq 0$; inverse exists$]$

$$\therefore\ A^{-1} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

$$= \begin{bmatrix} 2/5 & 1/5 & -1 \\ 1/5 & -2/5 & 1 \\ -3/5 & 1/5 & 1 \end{bmatrix}$$

$\therefore\ X = A^{-1}B$

$$= \underset{3\times 3}{\begin{bmatrix} 2/5 & 1/5 & -1 \\ 1/5 & -2/5 & 1 \\ -3/5 & 1/5 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}}$$

$$= \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}_{3\times1}$$

awards
∴ The ~~amount~~ to be given must amount to :

Rs 200 for sincerity
Rs 300 for truthfulness
Rs 400 for helpfulness.

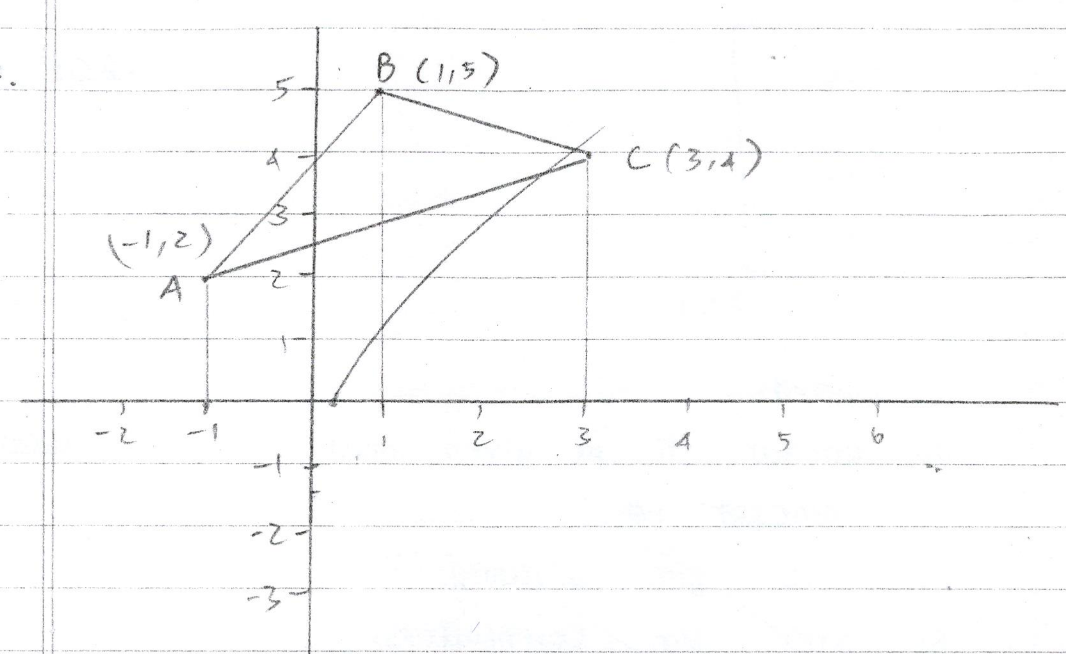
$$\frac{3200 + 2300}{5} - 900$$
$$= \frac{5500}{5} - 900$$
$$= 1100 - 900$$
$$= 200$$

$$\frac{1600 - 4600}{5} + 900$$
$$-\frac{3000}{5} + 900$$
$$-600 + 900 = 300$$
$$\frac{-4800 + 2300}{5} + 900$$
$$-\frac{2500}{5} + 900$$

* Another value which can be considered for awards is kindness. Kindness is an important quality in students that makes them good human beings. True friendship can also be rewarded by schools.

Hence, kindness and ~~true~~ friendship are the other two values that can be rewarded.

27.



$\frac{1}{2} \times \cancel{3 \times (3+4)}$

$\frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times (2+4)$

$2(6) = 12$

Equation of $\overleftrightarrow{AB}$ ÷ $(y-5) = \frac{3}{2}(x-1)$

$2y - 10 = 3x - 3$

$3x - 2y + 7 = 0$ $\qquad y = \frac{3x+7}{2}$

Eq. of $\overleftrightarrow{BC}$ ÷ $(y-5) = \frac{1}{-2}(x-1)$

$-2y + 10 = x - 1$

$x + 2y - 11 = 0$ $\qquad y = \frac{11-x}{2}$

Equation of AC ÷

$$(y-4) = \frac{\cancel{8}\,1}{\cancel{4}\,2}(x-3)$$

$$2y - 8 = x - 3$$

$$x - 2y + 5 = 0 \qquad \cancel{y = \frac{x+5}{2}}$$

(I) Area under AB ÷

$$A(I) = \int_{-1}^{1} y\,dx$$

$$= \int_{-1}^{1} \frac{1}{2}(3x+7)\,dx$$

$$= \frac{1}{2}\left(\frac{3x^2}{2} + 7x\right)\Big]_{-1}^{1}$$

$$= \frac{1}{2}\left[\cancel{\frac{3}{2}} + 7 - \cancel{\frac{3}{2}} + 7\right]$$

$$= 7 \text{ sq. units}$$

II

Area under BC :

$$A(II) = \int_1^3 y\,dx$$

$$= \int_1^3 \left(\frac{11-x}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{11x}{1} - \frac{x^2}{2}\right]_1^3$$

$$= \frac{1}{2}\left[33 - \frac{9}{2} - 11 + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[22 - \frac{8}{2}\right]$$

$$= \frac{1}{2}(22 - 4)$$

$$= 9 \text{ sq. units}$$

III

Area under AC :

$$A(III) = \int_{-1}^3 y\,dx$$

$$= \int_{-1}^{3} \frac{x+5}{2}\, dx$$

$$= \frac{1}{2} \int_{-1}^{3} (x+5)\, dx$$

$$= \frac{1}{2} \left(\frac{x^2}{2} + 5x\right)\Big]_{-1}^{3}$$

$$= \frac{1}{2} \left[\frac{9}{2} + 15 - \frac{1}{2} + 5\right]$$

$$= \frac{1}{2} \left[20 + 4\right] \quad \left(= \frac{24}{2} = 12\right)$$

$$= 12 \text{ sq. units}$$

$\therefore$ Required area of the triangle $= A(\mathrm{I}) + A(\mathrm{II}) - A(\mathrm{III})$

$$= (7 + 9 - 12) \text{ sq. units}$$

$$= (16 - 12) \text{ sq. units}$$

$$= 4 \text{ sq. units}$$

28. $I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

$I = \int \sqrt{\tan x}\,(1 + \cot x)\, dx$

$\sqrt{\tan x} = t$ $\qquad$ $\cot x = \frac{1}{t^2}$

$\tan x = t^2$

$\sec^2 x\, dx = 2t\,dt$ $\qquad$ $\sec^2 x = 1 + \tan^2 x$

$dx = \frac{2t\,dt}{1+t^4}$ $\qquad$ $= 1 + t^4$

$\therefore\ I = \int t\left(1 + \frac{1}{t^2}\right) \frac{2t\,dt}{1+t^4}$

$= \int t\left(\frac{t^2+1}{t^2}\right) \frac{2t}{1+t^4}\, dt$

$= \int \frac{2\cancel{t^2}\,(t^2+1)}{\cancel{t^2}\,(1+t^4)}\, dt$

$= 2\int \left(\frac{t^2+1}{t^4+1}\right) dt$

$(\div \text{ by } t^2)$

$$\therefore I = 2\int \frac{(1 + 1/t^2)}{(t^2 + 1/t^2)}\, dt$$

$$= 2\int \frac{(1 + 1/t^2)\, dt}{\left(t - \frac{1}{t}\right)^2 + 2}$$

Putting $t - \frac{1}{t} = u$

$$\left(1 + \frac{1}{t^2}\right) dt = du$$

$$\therefore I = 2\int \frac{du}{u^2 + 2}$$

$$\therefore I = 2\int \frac{du}{u^2 + (\sqrt{2})^2}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C_1$$

$$\therefore I = 2\int \frac{du}{u^2 + (\sqrt{2})^2}$$

$$= 2 \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$$

$$= \sqrt{2} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$$

$$= \sqrt{2} \tan^{-1}\left[\frac{\left(t - \frac{1}{t}\right)}{\sqrt{2}}\right] + c$$

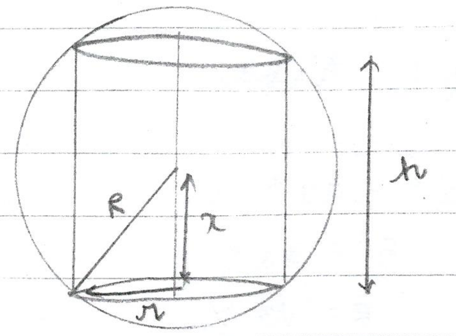
$$= \sqrt{2} \tan^{-1}\left[\frac{\sqrt{\tan x} - \frac{1}{\sqrt{\tan x}}}{\sqrt{2}}\right] + c$$

$$= \sqrt{2} \tan^{-1}\left[\frac{\tan x - 1}{\sqrt{2\tan x}}\right] + c$$

$$\therefore I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$$

$$= \sqrt{2} \tan^{-1}\left[\frac{\tan x - 1}{\sqrt{2\tan x}}\right] + c$$

29.



$h = 2x$

The radius of the sphere is R.

Let radius of cylinder be r, and its height be h.

~~Let x be the~~ $\cancel{x = \frac{h}{2}}$ $\rightarrow$ 1

$R^2 = x^2 + r^2$ $\quad r^2 = R^2 - x^2 \rightarrow$ 2.

Volume $= \pi r^2 h$

$= \cancel{\pi (R^2 - x^2)} \, 2x$ [from 1 and 2]

$\therefore V(x) = \pi (R^2 - x^2) \, 2x$

$V'(x) = 0$ ($\because$ Maximum volume)

$V'(x) = \pi(R^2 - x^2)\,2 + \pi(2x)[-2x]$

$= 0$

$\therefore \cancel{\pi}(R^2 - x^2)\cancel{2} + \cancel{\pi}(\cancel{2}x)(-2x) = 0$

$R^2 - x^2 - 2x^2 = 0$

$R^2 = 3x^2$

$R = \sqrt{3}\,x \qquad \therefore x = \frac{R}{\sqrt{3}} \rightarrow 3$

$r^2 = R^2 - x^2$

$= R^2 - \frac{R^2}{3}$

$= \frac{2R^2}{3}$

~~$\therefore$ volume~~ :

$\therefore h = 2x$ (from 1)

$\therefore h = 2\frac{R}{\sqrt{3}}$ (from 3)

Hence ; proved. // (that height is $\frac{2R}{\sqrt{3}}$) ~~times the~~

Second derivative test :-

$$V''(x) = 2\pi(-2x) + \pi(-8x)$$
$$= -4\pi x - 8\pi x$$
$$= -12\pi x$$
$$< 0$$

$\therefore$ $V''(x) < 0$

$\therefore$ It is a point of maxima.

$\therefore$ Maximum volume $= \pi r^2 h$

$$= \pi \frac{2R^2}{3} \times \frac{2R}{\sqrt{3}}$$

$$\therefore = \frac{4\pi}{3\sqrt{3}} R^3 \text{ cubic units.}$$

Section - B

11. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{\cancel{2}x}{a^2} - \frac{\cancel{2}y}{b^2}\frac{dy}{dx} = 0$

$\frac{x}{a^2} = \frac{y}{b^2}\left(\frac{dy}{dx}\right)$

$\therefore \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x}{y}$

~~Let~~ At any point (x_0, y_0) on the curve,

$\therefore \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x_0}{y_0}$

$\therefore$ At $(\sqrt{2}a, b)$ $\frac{dy}{dx} = \frac{b^{\cancel{2}}}{a^{\cancel{2}}}\frac{\sqrt{2}\cancel{a}}{\cancel{b}}$

$= \frac{\sqrt{2}b}{a}$

23.

y axis

Scale ⇒
x axis → 1cm → 5 units
y axis → 1cm → 5 units

$x \geq 0$; $y \geq 0$

hours

Let x be the no. of pieces of A and y be the no. of pieces of B to be manufactured

$9x + 12y \leq 180$

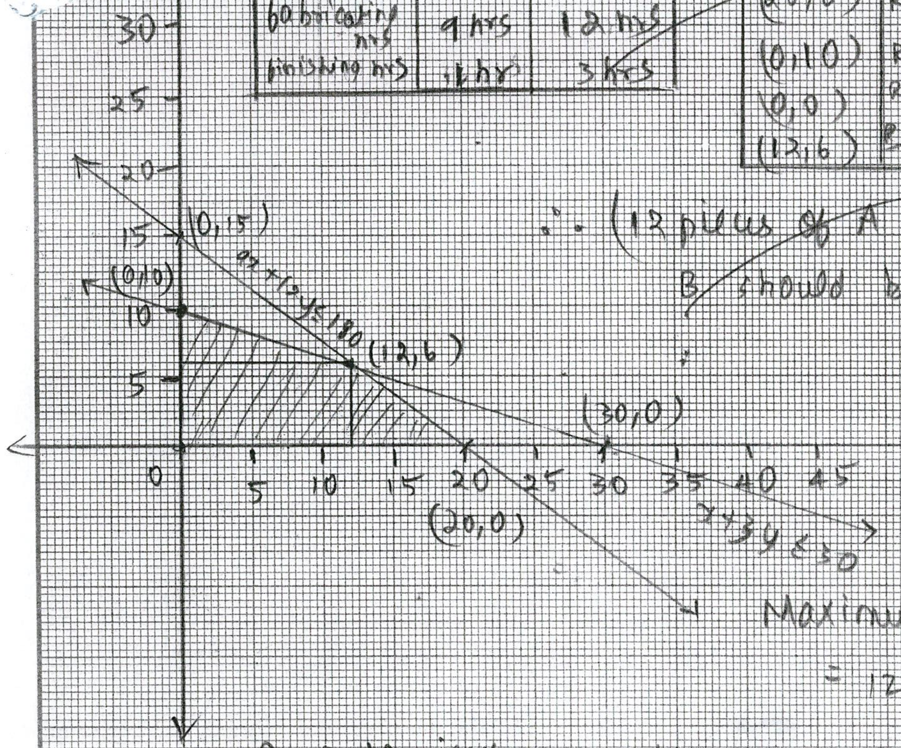
$x + 3y \leq 30$

$Z = 80x + 120y$

	x	y
Profit	RS 80	RS 120
fabricating hrs	9 hrs	12 hrs
finishing hrs	1 hr	3 hrs

corner point	Z
(20,0)	RS 1600
(0,10)	RS 1200
(0,0)	RS 0
(12,6)	RS 1680

∴ (12 pieces of A and 6 pieces of B should be manufactured)



Maximum profit
$= 12(80) + 6(120)$
$=$ RS 1680 per week

A → 12 pieces (per week)

B → 6 pieces (per week)

Max. profit = RS 1680 per week

E 2

E2

Equation of Tangent at $(\sqrt{2}a, b)$:

$$(y-b) = \frac{\sqrt{2}b}{a}(x-\sqrt{2}a)$$

$$(y-b)a = \sqrt{2}bx - 2ab$$

$\cancel{ay-by}$

$$ay - ab = \sqrt{2}bx - 2ab$$

$\cancel{ay}$

$$\sqrt{2}bx - ay = ab$$

$\because (\sqrt{2}a, b)$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{2a^2}{a^2} - \frac{b^2}{b^2} = 1$$

$(\div$ by $ab)$

$$\frac{\sqrt{2}\cancel{b}x}{a\cancel{b}} - \frac{\cancel{a}y}{\cancel{a}b} = 1$$

$$\therefore \frac{\sqrt{2}x}{a} - \frac{y}{b} = 1, \quad (\text{or}) \quad \sqrt{2}\,x\,b - ay = ab$$

(Eq. of tangent at $(\sqrt{2}a, b)$)

Equation of normal at $(\sqrt{2}a, b)$:

$$(y-b) = -\frac{dx}{dy}(x - \sqrt{2}a)$$

$$\frac{dy}{dx} = \frac{\sqrt{2}b}{a}$$

$$\therefore -\frac{dx}{dy} = -\frac{a}{b\sqrt{2}}$$

$$\therefore (y-b) = \frac{-a}{\sqrt{2}b}(x - \sqrt{2}a)$$

$$\sqrt{2}by - \sqrt{2}b^2 = -ax + \sqrt{2}a^2$$

$$ax + \sqrt{2}by = \sqrt{2}(a^2+b^2)$$

$$\frac{ax}{\sqrt{2}} + by = (a^2+b^2)$$ [equation of normal at $(\sqrt{2}a, b)$]

12. $I = \int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}\,dx$

$I = 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx \rightarrow (1)$

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\therefore I = 4\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$

$= 4\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx \rightarrow (2)$

Adding (1) and (2),

$2I = 4\int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}\,dx$

$\frac{\cancel{2}I}{2 \times \pi} = \int_0^{\pi} \frac{\sin x\,dx}{1+\cos^2 x}$

$$\frac{I}{2\pi} = \int_0^{\pi} \frac{\sin x \; dx}{1+\cos^2 x}$$

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \quad \left[\text{if } f(2a-x) = f(x)\right]$$

$$\therefore \frac{I}{2\pi} = 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$$

$$\frac{I}{4\pi} = \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$$

$\cos x = t$ if $x=0 \quad t=1$

$\therefore -\sin x\,dx = dt$ $x=\pi/2 \quad t=0$

$$\therefore \frac{I}{4\pi} = \int_1^{0} \frac{-dt}{1+t^2}$$

$$= \int_0^{1} \frac{dt}{1+t^2} \quad \left[\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right]$$

$$= \tan^{-1} t \Big]_0^1$$

$$\frac{I}{4\pi} = \frac{\pi}{4}$$

$$\therefore I = \pi^2$$

$$\therefore \int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}\,dx = \pi^2$$

13. $$y = Pe^{ax} + Qe^{bx} \rightarrow (1)$$

$$\frac{dy}{dx} = aPe^{ax} + bQe^{bx} \rightarrow (2)$$

From (1),

$$Pe^{ax} = y - Qe^{bx}$$

Substituting in (2),

$$\frac{dy}{dx} = a(y - Qe^{bx}) + bQe^{bx}$$

$$\frac{d^2y}{dx^2} = a^2Pe^{ax} + b^2Qe^{bx} \rightarrow (3)$$

Required to prove :

$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$$

Evaluating LHS,

$$LHS = \frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$$

$$= a^2Pe^{ax} + b^2Qe^{bx} - (a+b)\left[aPe^{ax} + bQe^{bx}\right]$$

[From ①, ②, and ③]

$$- ab\left(Pe^{ax} + Qe^{bx}\right)$$

$$= \cancel{a^2Pe^{ax}} + \cancel{b^2Qe^{bx}} - \cancel{a^2Pe^{ax}} - \cancel{b^2Qe^{bx}}$$

$$- \cancel{abQe^{bx}} - \cancel{abPe^{ax}} - \cancel{abPe^{ax}} - \cancel{abQe^{bx}}$$

$$= 0 \quad \text{(Cancelling all terms)}$$

$$RHS = 0$$

Hence, LHS = RHS

$\therefore \dfrac{d^2y}{dx^2} - (a+b)\dfrac{dy}{dx} + aby = 0$ ~~Hence~~ ; proved

14. $\tan^{-1}\left[\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \dfrac{\pi}{4} - \dfrac{\cos^{-1}x}{2}$

$LHS = \tan^{-1}\left[\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right]$

$x = \cos 2\theta \rightarrow (1)$

$LHS = \tan^{-1}\left[\dfrac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right]$

$= \tan^{-1}\left[\dfrac{\cancel{\sqrt{2}}\cos\theta - \cancel{\sqrt{2}}\sin\theta}{\cancel{\sqrt{2}}\cos\theta + \cancel{\sqrt{2}}\sin\theta}\right]$

$\because 1+\cos 2\theta = 2\cos^2\theta$

$1-\cos 2\theta = 2\sin^2\theta$

$$LHS = \tan^{-1}\left[\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right]$$

(÷ numerator and denominator by $\cos\theta$)

$$\therefore LHS = \tan^{-1}\left[\frac{1 - \tan\theta}{1 + \tan\theta}\right]$$

$$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$$

$$\therefore LHS = \tan^{-1}\left[\tan\left(\pi/4 - \theta\right)\right]$$

$$= \frac{\pi}{4} - \theta$$

$x = \cos 2\theta$

$\therefore 2\theta = \cos^{-1}x$

$\theta = (\cos^{-1}x) \times \frac{1}{2}$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

$= RHS$

$\therefore$ LHS = RHS

Hence ; proved.

15. $(1+x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}$

$$\frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\frac{dy}{dx} + Py = Q$$

(where P, Q are functions of x)

$$\Rightarrow \quad y e^{\int P dx} = \int Q e^{\int P dx} \cdot dx + c_1$$

integrating factor $\left\{ = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1}x} \right.$

$$\therefore \quad y e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x} \cdot dx + c \quad \rightarrow (1)$$

let I be $\int \frac{e^{\tan^{-1}x} \cdot e^{\tan^{-1}x}}{1+x^2}\, dx$

$\therefore\ I = \int \frac{e^{\tan^{-1}x} \cdot e^{\tan^{-1}x}}{1+x^2}\, dx$

Putting, $\tan^{-1}x = t$

$\therefore\ x = \tan t \Rightarrow 1+x^2 = \sec^2 t$

$\therefore\ dx = \sec^2 t\, dt$

$\therefore\ I = \int \frac{e^t \cdot e^t}{\cancel{\sec^2 t}} \cancel{\sec^2 t}\, dt$

$= \int e^{2t}\, dt$

$= \frac{1}{2} e^{2t} + c_2$

$= \frac{1}{2} e^{2\tan^{-1}x} + c_2$

substituting in ①,

$$y\, e^{\tan^{-1} x} = \frac{1}{2} e^{2\tan^{-1} x} + C_2 + C_1$$

$$= \frac{1}{2} e^{2\tan^{-1} x} + C \qquad (C = C_2 + C_1)$$

$$\therefore\ y = \frac{1}{2} \frac{e^{2\tan^{-1} x}}{e^{\tan^{-1} x}} + \frac{C}{e^{\tan^{-1} x}}$$

$$y = \frac{1}{2} e^{\tan^{-1} x} + \frac{C}{e^{\tan^{-1} x}}$$

16. $\overrightarrow{AB} = \cancel{4}\hat{i} - \hat{j} - \hat{k} - 4\hat{i} - 5\hat{j} - \hat{k}$

$= -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$

$\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are coinitial vectors.

For $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ to be coplanar,

$$\vec{AB} \cdot [\vec{AC} \times \vec{AD}] = 0$$

$\therefore$ ~~[AB~~

$$\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = \vec{AB} \cdot [\vec{AC} \times \vec{AD}]$$

$$= -4(12+3) + 6(-3+24) - 2(\overset{1+32}{\cancel{-3} + \cancel{24}})$$

$$= -4(15) + 6(21) - 2(33)$$

$$= -60 + 126 - 66$$

$$= 126 - 126$$

$$= 0$$

$\therefore \quad \vec{AB} \cdot [\vec{AC} \times \vec{AD}] = 0$

$\therefore$ ~~$\vec{A}$, $\vec{B}$~~, A, B, C and D are coplanar points.

Hence, proved.

17. $f(x) = x^2 + 2$

$f : R \to R$

$g(x) = \dfrac{x}{x-1} \quad (x \neq 1)$

$g : R \to R$

$fog(x) = f[g(x)]$ fog exists.

$= f\left(\dfrac{x}{x-1}\right)$ ($\because$ Range of $g(x)$ and domain of $f(x)$ coincide)

$= \left(\dfrac{x}{x-1}\right)^2 + 2$ $\therefore$ fog exists.

$= \dfrac{x^2}{(x-1)^2} + 2$

$= \dfrac{x^2 + 2(x-1)^2}{(x-1)^2}$

$= \dfrac{x^2 + 2x^2 + 2 - 4x}{(x-1)^2} = \dfrac{3x^2 - 4x + 2}{(x-1)^2} \quad (x \neq 1)$

$\therefore\ fog(2) = \left(\frac{2}{2-1}\right)^2 + 2$

$= 2^2 + 2$

$= 4 + 2$

$= 6$

$gof(x) = g(f(x))$

$= g(x^2+2)$

$= \frac{x^2+2}{x^2+2-1}$

$= \frac{x^2+2}{x^2+1}$

$gof(x)$ exists

($\because$ range of $f(x)$ and domain of $g(x)$ coincide)

$\therefore\ gof(x)$ exists

~~go~~

$\therefore\ gof(-3) = \frac{9+2}{9+1}$

$= \frac{11}{10}$

18. $p = 3q$ $\quad$ $p \to$ success probability

$q \to$ failure probability

$p + q = 1$

$\therefore 3q + q = 1$

$q = 1/4$

$p = 3/4$

$P(3) \to$ prob. of 3 succ.

$P(4) \to$ prob. of 4 succ.

$P(5) \to$ prob. of 5 succ.

$n = 5$

probability of atleast 3 successes $= [P(3) + P(4) + P(5)]$

~~$P(3) = 5C_3 \; \frac{3}{2}$~~

$P(x) = nC_x \; p^x \; q^{n-x}$

$\therefore P(3) = 5C_3 \; (3/4)^3 \left(\frac{1}{4}\right)^2$ $\qquad$ $P(5) = 5C_5 \; (3/4)^5 \; (1/4)^0$

$P(4) = 5C_4 \; (3/4)^4 \left(\frac{1}{4}\right)^1$

$\therefore P(3) + P(4) + P(5) = {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2$

$+ {}^5C_4 (3/4)^4 (1/4)$

$+ {}^5C_5 (3/4)^5 (1/4)^0$

$= 10 \left(\frac{3^3}{4^5}\right) + 5 \frac{(3^4)}{4^5} + \left(\frac{3^5}{4^5}\right)$

$= \frac{270 + 405 + 243}{4^5}$

$= \frac{918}{4^5} = \frac{409}{2 \times 4^4} = \frac{409}{512}$ //

$= \frac{459}{512}$ //

$\frac{5 \times 4 \times 3}{1 \times 2 \times 3}$

$81 \times 5 = 405$

$81 \times 3 = 243$

$270 + 405 + 243 = 918$

$64 \times 4 = 256$

19. $$LHS = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow LHS = \begin{vmatrix} 2a+2b+2c & c+a & a+b \\ 2p+2q+2r & r+p & p+q \\ 2x+2y+2z & z+x & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

$C_2 \to C_2 - C_1 \qquad C_3 \to C_3 - C_1$

$$\therefore LHS = 2\begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$\therefore LHS = 2\begin{vmatrix} a+\cancel{b}+\cancel{c}-\cancel{b}-\cancel{c} & -b & -c \\ p+\cancel{q}+\cancel{r}-\cancel{q}-\cancel{r} & -q & -r \\ x+\cancel{y}+\cancel{z}-\cancel{y}-\cancel{z} & -y & -z \end{vmatrix}$$

$$= 2(-1)(-1)\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

(Taking (-1) common from C_2 and C_3)

$$= 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

Hence ; LHS = RHS

Hence ; proved

20. $x = a \sin 2t \, (1 + \cos 2t)$

$$\frac{dx}{dt} = a \left[\sin 2t \, (-2 \sin 2t) + (1 + \cos 2t) \, 2 \cos 2t \right]$$

$$= a \left[-2 \sin^2 2t + 2 \cos 2t + 2 \cos^2 2t \right]$$

$$= 2a \left[\cos^2 2t - \sin^2 2t + \cos 2t \right] = 2a \left[\cos 4t + \cos 2t \right]$$

$y = b \cos 2t \, (1 - \cos 2t)$

$$\frac{dy}{dt} = b \left[\cos 2t \, (2 \sin 2t) + (1 - \cos 2t)(-2 \sin 2t) \right]$$

$$= b \left[2 \cos 2t \sin 2t + 2 \cos 2t \sin 2t - 2 \sin 2t \right]$$

$$= b \left[2 \sin 4t - 2 \sin 2t \right]$$

$$= 2b \, (\sin 4t - \sin 2t)$$

$$\therefore \frac{dy}{dx} = \frac{2b}{2a} \frac{(\sin 4t - \sin 2t)}{(\cos 4t + \cos 2t)}$$

$$= \frac{b}{a} \frac{\cancel{2}(\sin t)\cos(3t)}{\cancel{2}\cos 3t\,\cancel{\sin t}(\cos t)}$$

$$= \frac{b}{a} \tan t$$

$$\therefore \left.\frac{dy}{dx}\right]_{t=\pi/4} = \frac{b}{a}\tan \pi/4$$

$$= b/a$$

Hence; proved

21. $$x(1+y^2)\,dx = y(1+x^2)\,dy$$

$$\frac{dy}{dx} = \frac{x(1+y^2)}{y(1+x^2)}$$

$$\therefore \int dy \frac{(y)}{1+y^2} = \int \frac{x\,dx}{1+x^2}$$

$1+y^2 = t \qquad 1+x^2 = u$

$2y\,dy = dt \qquad 2x\,dx = du$

$y\,dy = \frac{dt}{2} \qquad x\,dx = \frac{du}{2}$

$$\therefore \int \frac{dt}{\cancel{2}(t)} = \int \frac{du}{\cancel{2}(u)}$$

$$\therefore \log(t) = \log(u) + c$$

$$\therefore \log(1+y^2) = \log(1+x^2) + c$$

$\cancel{\therefore \; 1+y^2 =}$

$$\therefore \log\left(\frac{1+y^2}{1+x^2}\right) = c$$

when $y=1 \quad x=0 \Rightarrow \log 2 = c$

$\therefore \log\left(\frac{1+y^2}{1+x^2}\right) = \log 2$

$\therefore 1+y^2 = 2(1+x^2)$ //

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22. $L_1 : \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$

$L_2 : \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ $\left(\because \text{Eq. of line} : \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}\right)$

Let the eq. of the line (req.) be $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$

$a+2b+3c=0$ ($\because$ the req. line is $\perp$R to both L_1 and L_2)

$-3a+2b+5c=0$

$4a-2c=0$

$4a=2c$

$2a=c \rightarrow (1)$

$\therefore a+2b+6a=0 \rightarrow (2)$

$2b=-7a$ $\therefore b = \frac{-7a}{2}$

0901

Fictitious Roll No.
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$$\therefore \quad \frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$$

$$\Rightarrow \quad \frac{x-2}{\cancel{a}} = \frac{y-b}{-\frac{7\cancel{a}}{2}} = \frac{z-3}{2\cancel{a}} \qquad \text{(from ① and ②)}$$

$$\Rightarrow \quad \frac{x-2}{1} = \frac{y-1}{-7/2} = \frac{z-3}{2}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4} \qquad \text{(Cartesian equation)}$$

$$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

(Vector equation)

Section- A

1. $x - y = -1$

$2x - y = 0$

$y = 2x$

$x - 2x = -1$

$x = 1$

$\therefore y = 2$

$\therefore x + y = 3$

2. $12x + 14 = 3x - 42$

$12x + 14 = -10$

$12x = -24$

$x = -2$

3. $f(x) = \int_0^x t \sin t \, dt$

$f'(x) = x \sin x$

4. $x + 2y = 8$

Range $= \{1, 2, 3\}$

5. $\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi/4$

$x + y = 1 - xy$

$x + y + xy = 1$

6. $7A - I^3 = A^3 - 3A^2I - 3AI^2$

$= \cancel{7}A - I - \cancel{A} - \cancel{3}A - \cancel{3}A$ $\left[\because A^2 = A,\ A^3 = A\right]$

$= -I$

$\therefore 7A - (I+A)^3 = -I$

7. $\frac{k}{3} = -kp$

$\therefore k = -3 \quad \therefore p = -1/3$

8. $\dfrac{x-3}{-5} = \dfrac{y+4}{7} = \dfrac{z-3}{2}$

$\therefore \vec{r} = (+3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$

9. $\log x = t$ $\qquad \dfrac{dx}{x} = dt$

$\therefore \int_1^2 \dfrac{1}{t} dt$

$= \log t \Big]_1^2$

$= \log 2$

10. $l^2 + m^2 + n^2 = 1$

$\dfrac{1}{2} + 0 + n^2 = 1$ $\qquad \therefore \vec{a} = \cancel{5\sqrt{2}}$

$n^2 = 1/2$

$\therefore \vec{a} = 5\sqrt{2}\frac{1}{\sqrt{2}}\hat{i} + 0 + 5\sqrt{2}\frac{1}{\sqrt{2}}\hat{k}$

$= 5\hat{i} + 5\hat{k}$

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